$w = \Delta_0 - \Delta$

Displacements: $(w, s)$

Contact Forces: $(F_n \text{ and } F_t)$

$F_n \sim C_n w^{3/2}$

$F_t \sim C_t w^{1/2} s$
Comparison of Simulation, Experiment and Theory

Effective Medium Theory:

\[ K \sim C_n (1-\phi)^{2/3} Z^{2/3} P^{1/3} \]
\[ \mu \sim [C_n + 3/2 C_t] (1-\phi)^{2/3} Z^{2/3} P^{1/3} \]

- \( P \) = Pressure
- \( Z \) = Average number of contacts
- \( \phi \) = Porosity
Problems with the Shear Modulus

\[ \frac{\mu}{P^{1/3}} \]

\[ Z (\text{Number of Contacts}) \sim 6 \]

Pressure (MPa)

EMT (Corrected)
MD
Domenico
Yin
Gland & St. Germain
Grain Rotations and Effective Medium Theory

Let the Transverse Force Vary:
\[ F_t \sim \alpha C_t w^{1/2} s \]

Calculation of Moduli:
Begin with affine motion (all grains move together)

Loss of rigidity at random close packing
Similar to melting transition in emulsions and foams
Evolution of Force Chains

$\sigma_{zz} = 100$ KPa

$F_{\text{min}} = 0.00590$

$F_{\text{max}} = 0.01490$
$\sigma_{zz} = 500$ KPa

$F_{\text{min}} = 0.01680$

$F_{\text{max}} = 0.03780$
\( \sigma_{zz} = 1.5 \text{ MPa} \)

\( F_{\text{min}} = 0.04110 \)
\( F_{\text{max}} = 0.09590 \)
\( \sigma_{zz} = 9 \text{ MPa} \)
\[ \sigma_{zz} = 20 \text{ MPa} \]

\[ F_{\text{min}} = 0.56800 \]
\[ F_{\text{max}} = 1.28000 \]
$\sigma_{zz} = 30 \text{ MPa}$

100 KPa

6 MPa

30 MPa
Combined Cementation and Pressure

A Model for Weakly Consolidated Sandstones

Inter-grain force controlled by stress and cement

Contacts “Break” (slide) when a random threshold is reached

Plona and Cook
How Close is the Rock to Failure?

Wellbore Stability
Sand Control

Continuum Models
SDR-Acoustics
SCR-Geomechanics

Microscopic Rock Models
(Weakly Consolidated Sandstones)

Objective: Model Based Interpretation