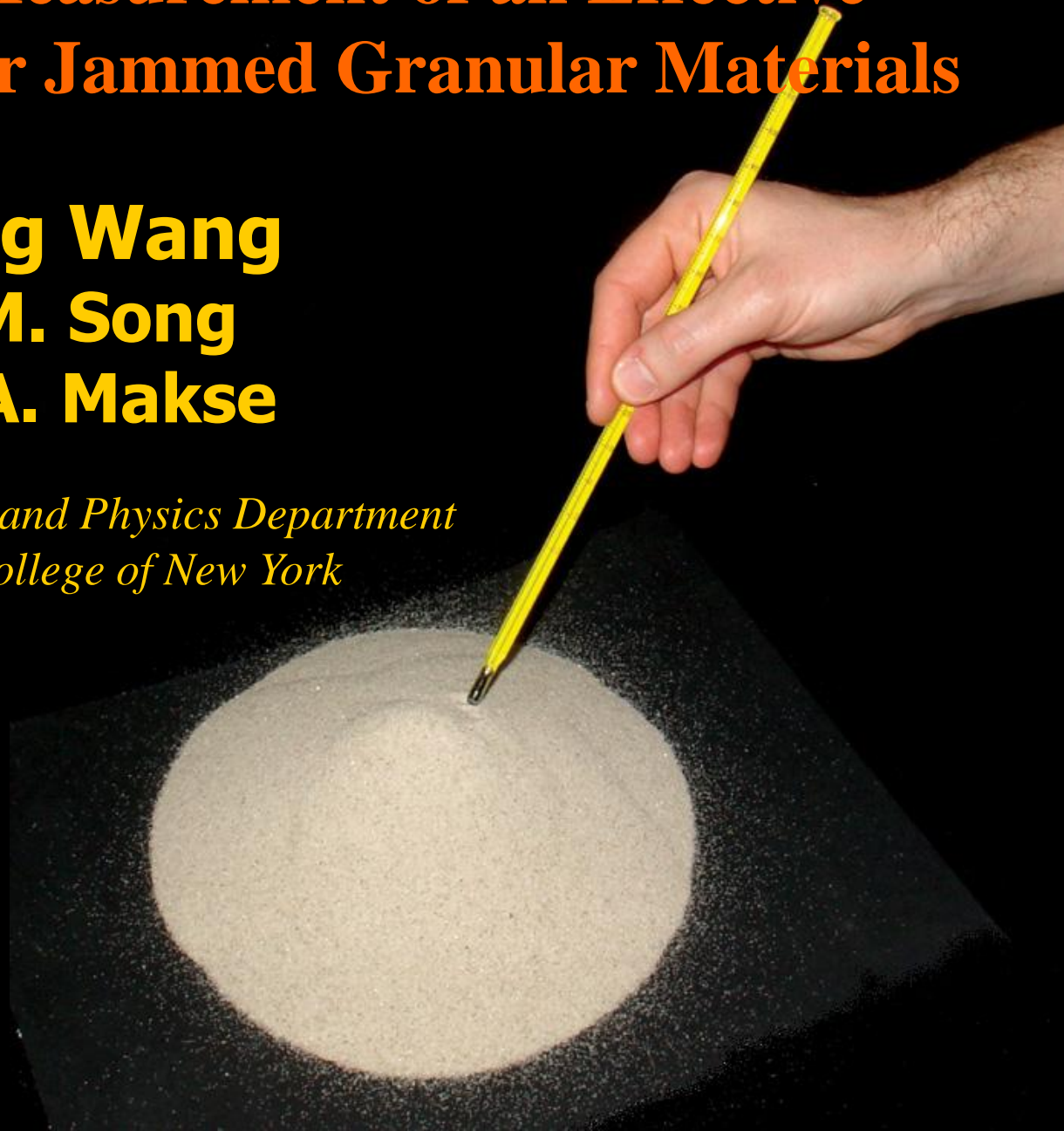


# Experimental Measurement of an Effective Temperature for Jammed Granular Materials

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# Dynamical Definition of Effective Temperature

We define  $T_{\text{eff}}$  using a FDT theorem

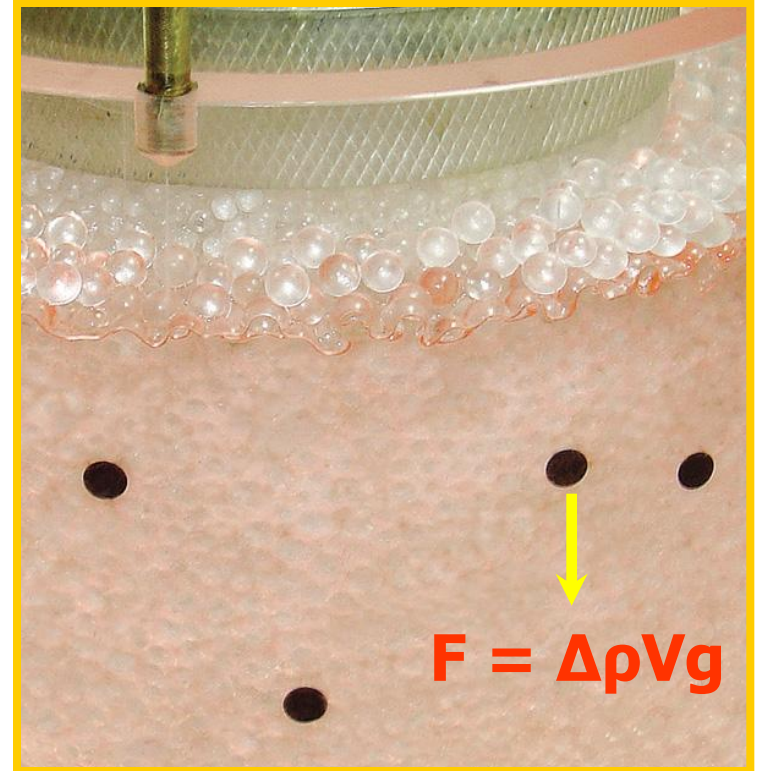
Follow add tracer in a slow granular shear flow

- Correlation Function: "Brownian motion"

$$\langle |x(t) - x(0)|^2 \rangle \sim 2Dt \quad (D = \text{diffusivity})$$

- Response function: gently pull with an external force  $F$

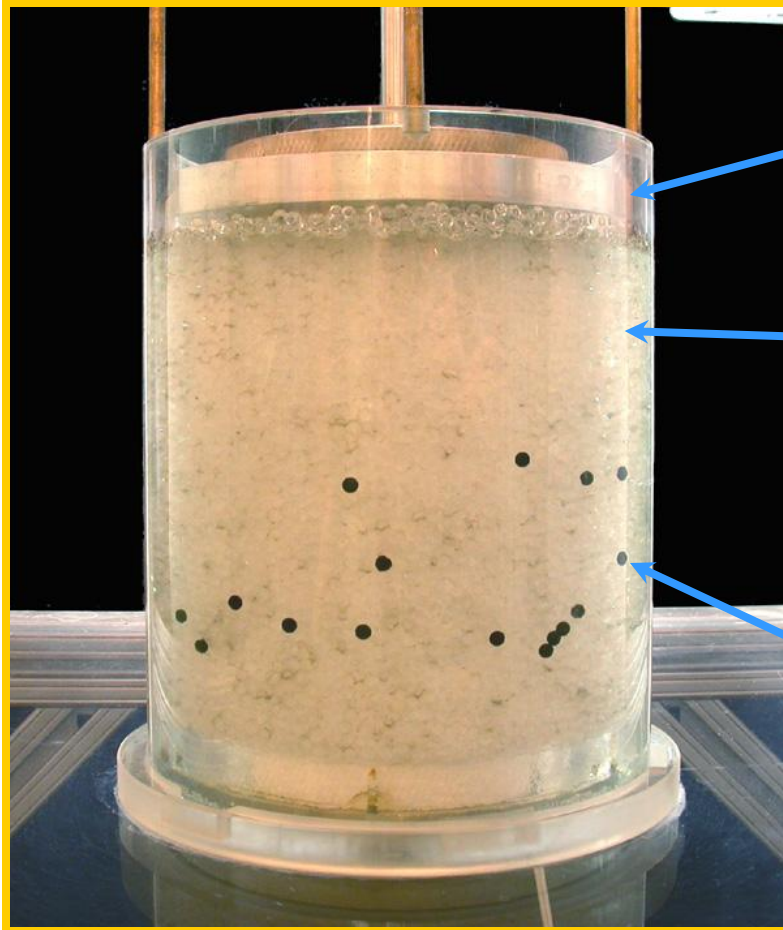
$$\langle |x(t) - x(0)| \rangle \sim M F t \quad (M = \text{mobility})$$



- Stokes-Einstein relation:

$$\langle |x(t) - x(0)|^2 \rangle = 2 T_{\text{eff}} \langle |x(t) - x(0)| \rangle / F$$

# Experimental set-up to measure the effective temperature in quasi-static granular materials

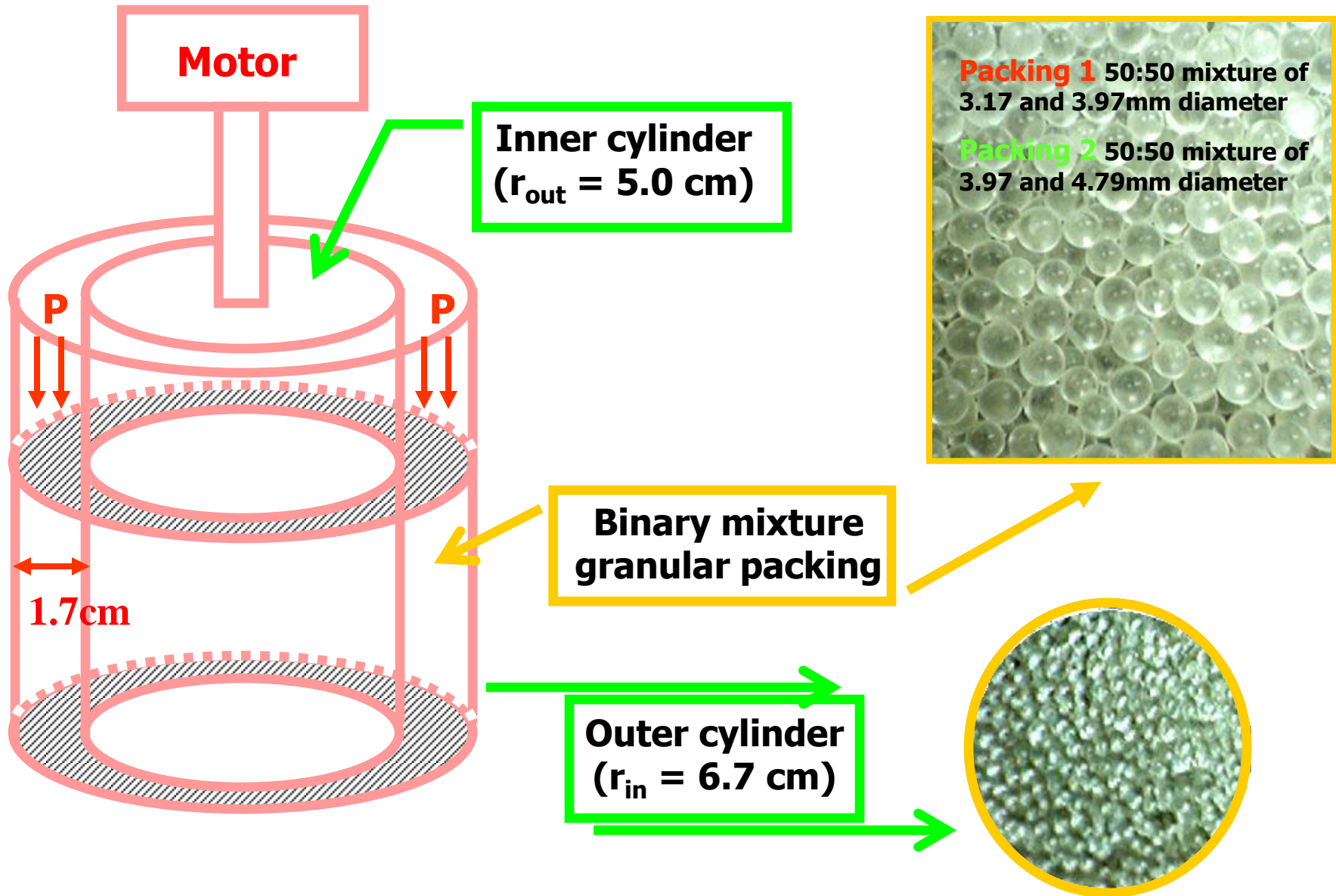


System consists of:

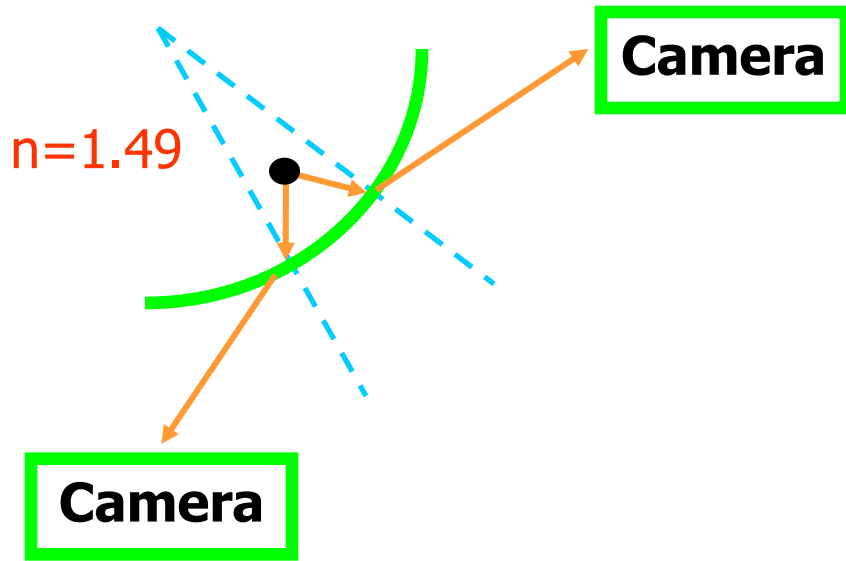
- Slowly sheared **Couette Cell** (0.01/s) at constant pressure(0.4kPa).
- Background **PMMA (acrylic)** particles (50:50 mixture of 3.17 and 3.97mm diameter) in a refractive index and density matching **solution**.
- **20 tracers** with a different density from the background particles.

Tracer **trajectories** are tracked and used to determine the effective temperature.

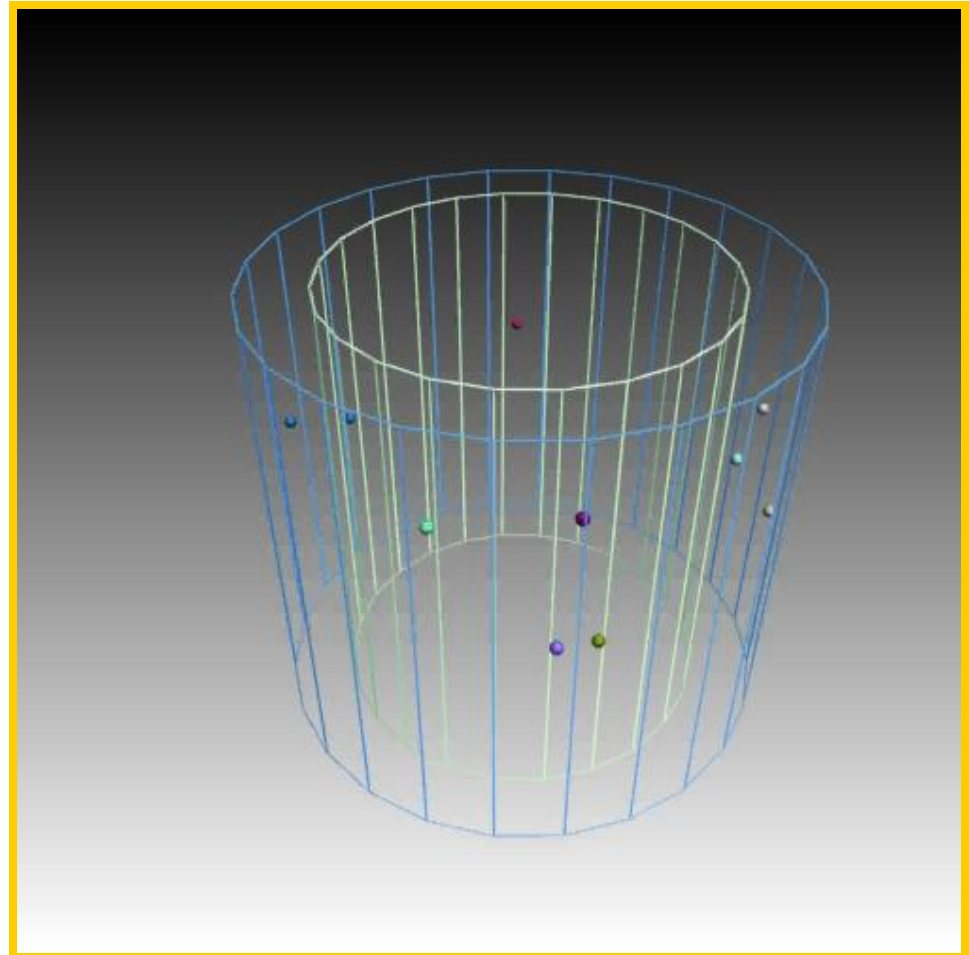
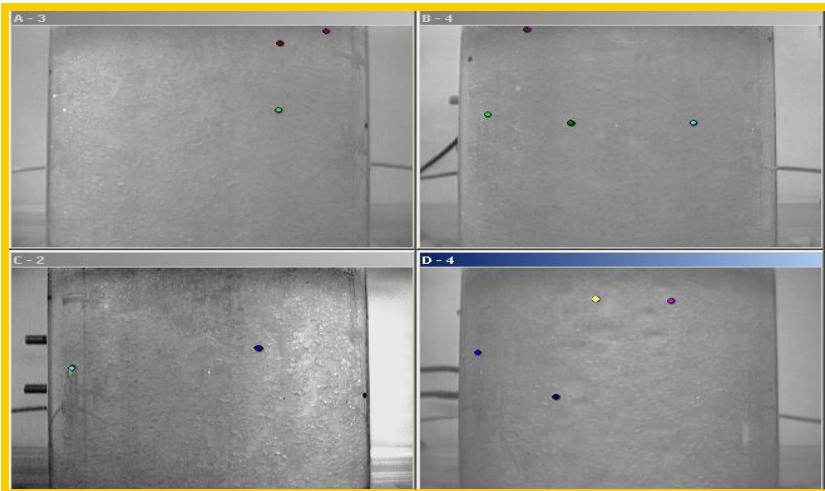
# Sketch of Experimental Set-Up



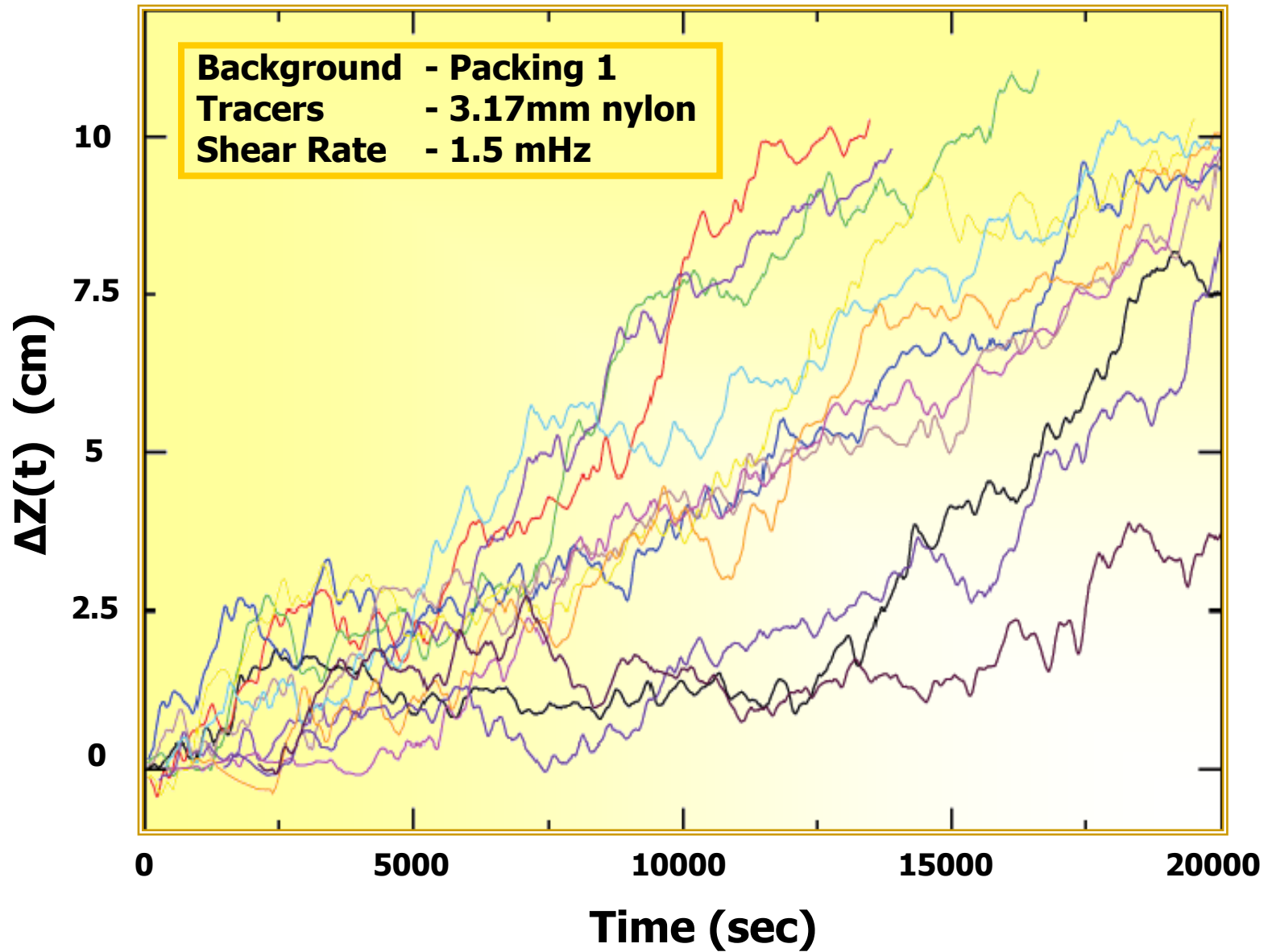
# Technique of tracking tracers



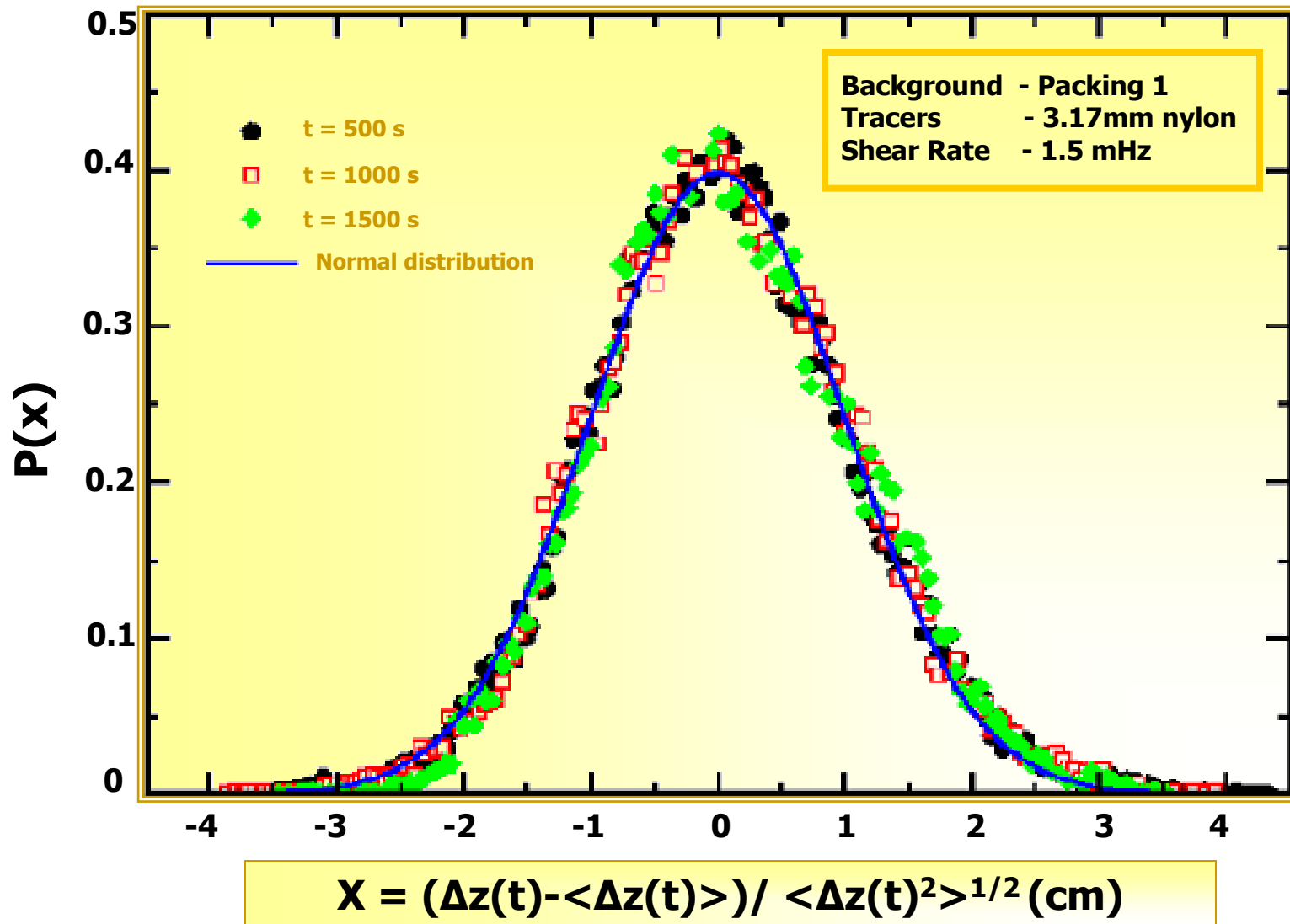
120 x realtime



# Trajectories of tracers

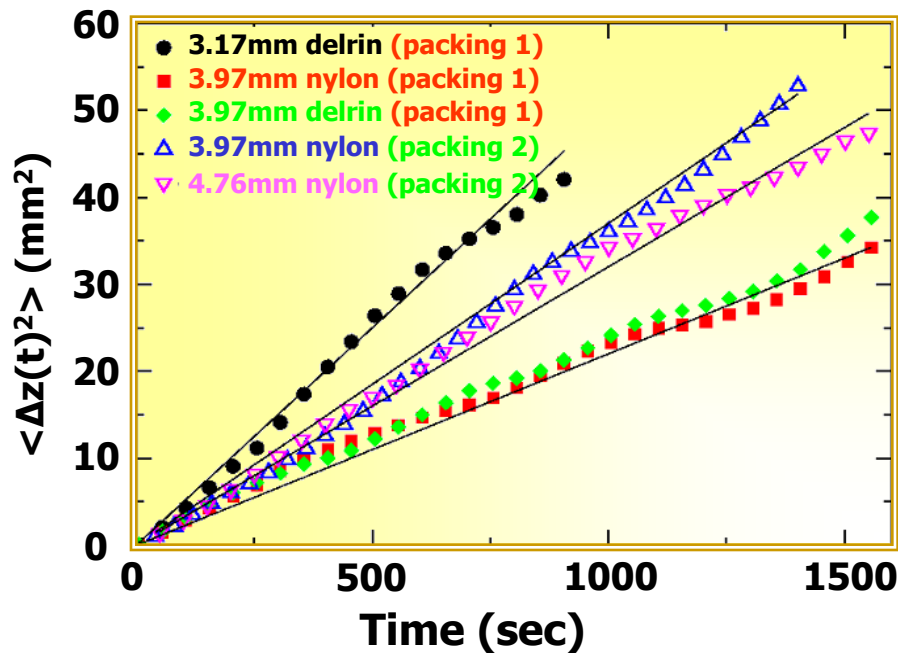


# Gaussian distribution of tracer displacements at different times

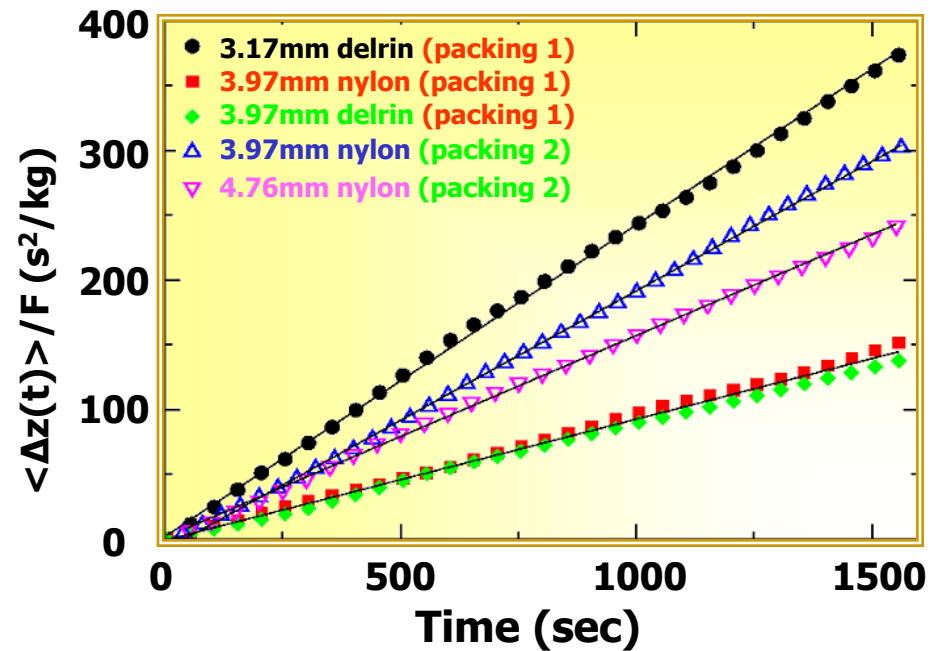


# Diffusion and mobility for different tracers and packings

## Diffusion



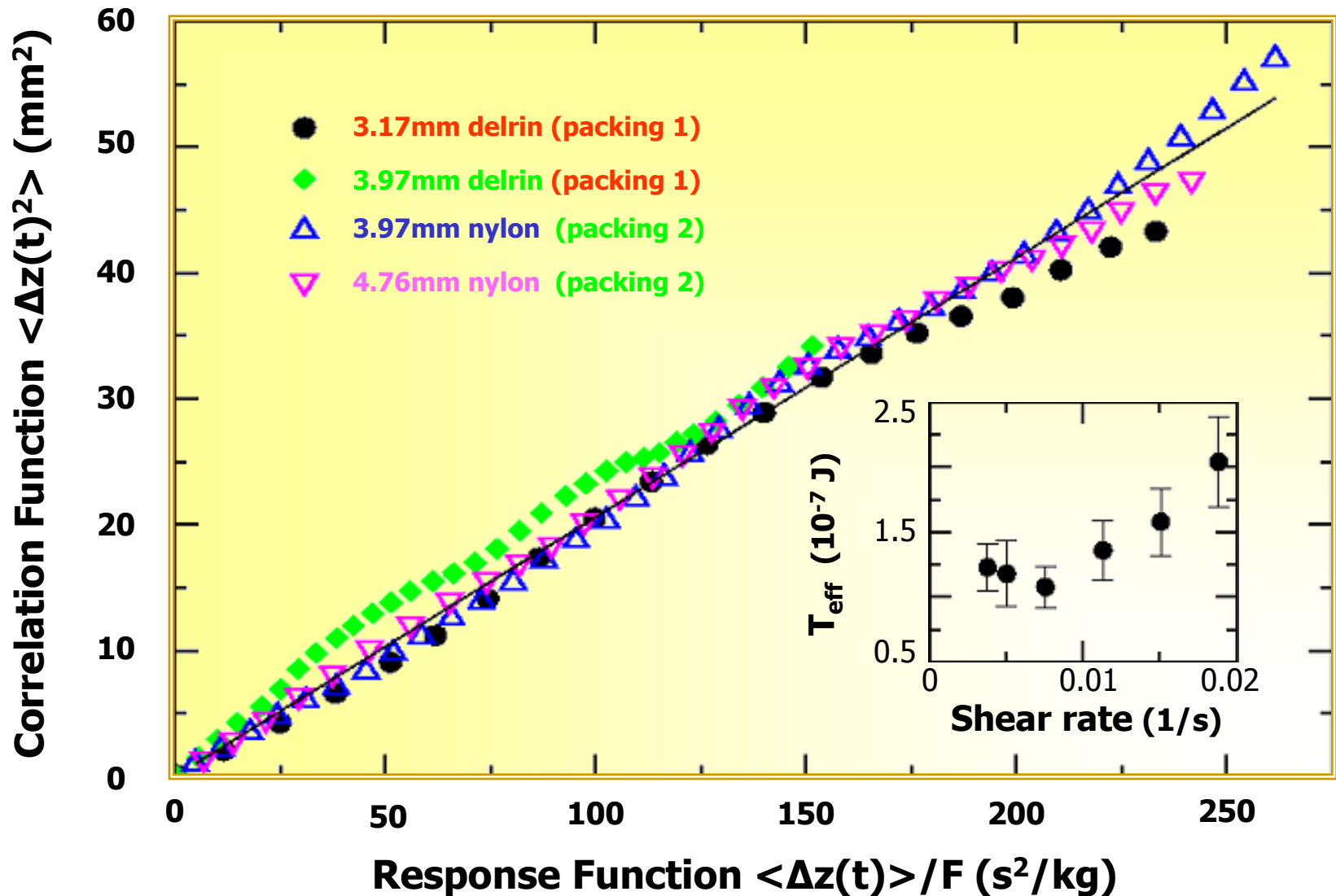
## Mobility



- Diffusivity and Mobility **increase** with decreasing tracers size.
- Mobility is independent of drag force: **linear response regime**



# Same effective temperature for tracers with different sizes and different densities

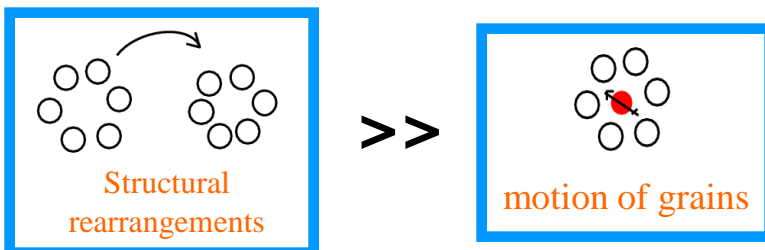


$$T_{\text{eff}} > T_{\text{kin}} > T_{\text{sun}}$$

Temperature of the sun  
 $\sim 1-2 \times 10^6 \text{ K}$

$$T_{\text{eff}} \sim 8 \times 10^{14} \text{ K}$$
$$T_{\text{kin}} \sim 3 \times 10^{13} \text{ K (3mm delrin)}$$
$$5 \times 10^{13} \text{ K (4mm delrin)}$$

$T_{\text{eff}} > T_{\text{kin}}$ :  
average energy to rearrange few grains  
is much higher than their kinetic energy.



# Conclusions

1. We have tested **the existence of  $T_{\text{eff}}$**  for various particle sizes and densities in a slowly sheared granular material very close to jamming.  
(*Song et al. , PNAS, 2004*)
2. All tracers **independently of their characteristics** equilibrate at the same  $T_{\text{eff}}$ , which is given by packing density.
3.  $T_{\text{eff}}$  is **a state variable** for the nearly jammed systems
4. Question: has  $T_{\text{eff}}$  a physical thermodynamic meaning?
5. Further work: test whether  $T_{\text{eff}}$  remains the same for different types of driving (tapping or shaking), and for different observables.

