Experimental Measurement of an Effective Temperature for Jammed Granular Materials

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Dynamical Definition of Effective Temperature

We define $T_{\text{eff}}$ using a FDT theorem

Follow add tracer in a slow granular shear flow

- Correlation Function: “Brownian motion”
  \[
  \langle |x(t)-x(0)|^2 \rangle \sim 2Dt \quad (D = \text{diffusivity})
  \]

- Response function: gently pull with an external force $F$
  \[
  \langle |x(t)-x(0)| \rangle \sim M F t \quad (M = \text{mobility})
  \]

- Stokes-Einstein relation:
  \[
  \langle |x(t)-x(0)|^2 \rangle = 2 T_{\text{eff}} \langle |x(t)-x(0)| \rangle / F
  \]
Experimental set-up to measure the effective temperature in quasi-static granular materials

System consists of:
- Slowly sheared **Couette Cell** (0.01/s) at constant pressure (0.4kPa).
- Background **PMMA (acrylic)** particles (50:50 mixture of 3.17 and 3.97mm diameter) in a refractive index and density matching **solution**.
- **20 tracers** with a different density from the background particles.

Tracer **trajectories** are tracked and used to determine the effective temperature.
Motor

Sketch of Experimental Set-Up

Inner cylinder ($r_{\text{out}} = 5.0 \text{ cm}$)

Binary mixture granular packing

Outer cylinder ($r_{\text{in}} = 6.7 \text{ cm}$)

Packing 1 50:50 mixture of 3.17 and 3.97 mm diameter

Packing 2 50:50 mixture of 3.97 and 4.79 mm diameter
Technique of tracking tracers

$n = 1.49$

120 x realtime
Trajectories of tracers

Background - Packing 1
Tracers - 3.17mm nylon
Shear Rate - 1.5 mHz
Gaussian distribution of tracer displacements at different times

\[ X = \frac{\Delta z(t) - \langle \Delta z(t) \rangle}{\langle \Delta z(t)^2 \rangle^{1/2}} \text{(cm)} \]
Diffusion and mobility for different tracers and packings

- Diffusivity and Mobility **increase** with decreasing tracers size.
- Mobility is independent of drag force: **linear response regime**
Same effective temperature for tracers with different sizes and different densities

Correlation Function $\langle \Delta z(t)^2 \rangle$ (mm$^2$)

Response Function $\langle \Delta z(t) \rangle/F$ (s$^2$/kg)

- 3.17mm delrin (packing 1)
- 3.97mm delrin (packing 1)
- 3.97mm nylon (packing 2)
- 4.76mm nylon (packing 2)
Temperature of the sun
\[ \sim 1-2 \times 10^6 \text{ K} \]

\[ T_{\text{eff}} \approx 8 \times 10^{14} \text{ K} \]
\[ T_{\text{kin}} \approx 3 \times 10^{13} \text{ K} \text{ (3mm delrin)} \]
\[ 5 \times 10^{13} \text{ K} \text{ (4mm delrin)} \]

\[ T_{\text{eff}} > T_{\text{kin}} > T_{\text{sun}} \]

\( T_{\text{eff}} > T_{\text{kin}} \): average energy to rearrange few grains is much higher than their kinetic energy.

Structural rearrangements

motion of grains
Conclusions

1. We have tested the existence of $T_{\text{eff}}$ for various particle sizes and densities in a slowly sheared granular material very close to jamming. (Song et al., PNAS, 2004)

2. All tracers independently of their characteristics equilibrate at the same $T_{\text{eff}}$, which is given by packing density.

3. $T_{\text{eff}}$ is a state variable for the nearly jammed systems.

4. Question: has $T_{\text{eff}}$ a physical thermodynamic meaning?

5. Further work: test whether $T_{\text{eff}}$ remains the same for different types of driving (tapping or shaking), and for different observables.